GCSE Maths - Algebra

Solving Quadratic Equations

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of questions involving solving quadratics. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Solve the following quadratic equation by factorisation:

$$\boldsymbol{x^2 + 11x + 24 = 0}$$

Step 1: We can start by writing down the brackets in the form of $(x + p)(x + q) = 0$. Leave out the value of p and q first since we will fill this out later. Ensure that the x in the bracket, when multiplied with each other, gives the quadratic part of the equation.

For instance, in this example, our quadratic term is x^2.
Hence, we can write our equation in the form:

$$(x + p)(x + q) = 0$$

Step 2: Comparing the equation to the form $ax^2 + bx + c = 0$, list all the possible factor pairs for c. Factor pair means a pair of integers which when multiplied together, equal c.

$$ax^2 + bx + c = x^2 + 11x + 24$$

In this case, $c = 24$.
The factor pairs for 24 *are:*

$$1 \times 24$$
$$2 \times 12$$
$$3 \times 8$$
$$4 \times 6$$

Step 3: Identify factor pairs which add together to give b.

$$ax^2 + bx + c = x^2 + 11x + 24$$

In this case, $b = 11$.

$$1 + 24 \neq 11$$
$$2 + 12 \neq 11$$
$$\boldsymbol{3 + 8 = 11}$$
$$4 + 6 \neq 11$$

So, the correct factor pair is 3 *and* 8.

Step 4: Substitute the correct factor pair as p and q in the bracket form that was formed in **Step 1**.

$$(x + p)(x + q) = 0$$
$$(x + 3)(x + 8) = 0$$

Step 5: Solve the equation by setting each bracket equal to 0.

$$x + 3 = 0 \quad \textit{or} \quad x + 8 = 0$$
$$x = -3 \qquad\qquad x = -8$$

Hence, the possible solutions for x are $\boldsymbol{x = -3}$ and $\boldsymbol{x = -8}$.

Guided Example

Solve the following quadratic equation by factorisation:

$$x^2 + 18x + 32 = 0$$

Step 1: We can start by writing down the brackets in the form of $(x+p)(x+q) = 0$. Leave out the value of p and q first since we will fill this out later. Ensure that the x in the bracket, when multiplied with each other, gives the quadratic part of the equation.

$$(x+p)(x+q) = 0$$

$x \times x = x^2$ ✓

Step 2: Comparing the equation to the form $ax^2 + bx + c = 0$, list all the possible factor pairs for c. Factor pair means a pair of integers which when multiplied together, equal c.

$$\begin{aligned} 32 &= 8 \times 4 \\ &= 2 \times 16 \\ &= 1 \times 32 \end{aligned}$$

Step 3: Identify which factor pair adds to give b.

$8 + 4 = 12$ ✗

$2 + 16 = 18$ ✓ these are the correct factors.

$1 + 32 = 33$ ✗

Step 4: Substitute the correct factor pair as p and q in the bracket form that was formed in **Step 1**.

$$(x+16)(x+2) = 0$$

Substitute for p and q.

Step 5: Solve the equation by setting each bracket equal to 0.

$x + 16 = 0$ $\quad -16$

$x = -16$

$x + 2 = 0$ $\quad -2$

$x = -2$

Therefore $x = -16$ and $x = -2$.

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

1. Solve the following quadratic equations by factorisation:

a) $t^2 - 6t - 27 = 0$

$-27 = -9 \times 3 \rightarrow \boxed{-9+3} = -6$ ✓

$= -3 \times 9 \rightarrow -3+9 = 6$ ✗

$= -27 \times 1 \rightarrow -27+1 = -26$ ✗

$= -1 \times 27 \rightarrow -1+27 = 26$ ✗

$(t+p)(t+q) = 0$

$(t+3)(t-9) = 0$

$t+3=0$ $\quad$ $t-9=0$

$t=-3$ $\quad$ $t=9$

b) $y^2 + 18y + 45 = 0$

$45 = 3 \times 15 \rightarrow \boxed{3+15} = 18$ ✓

$= 1 \times 45 \rightarrow 1+45 = 46$ ✗

$= 9 \times 5 \rightarrow 9+5 = 14$ ✗

$(y+q)(y+p) = 0$

$(y+3)(y+15) = 0$

$y+3=0$ $\quad$ $y+15=0$

$y=-3$ $\quad$ $y=-15$

c) $x^2 = 14x - 49$

$-14x, +49$ $\quad$ $-14x, +49$

$x^2 - 14x + 49 = 0$

$49 = 7 \times 7 \rightarrow 7+7 = 14$ ✗

$= -7 \times -7 \rightarrow \boxed{-7+-7} = -14$ ✓

$= 49 \times 1 \rightarrow 49+1 = 50$ ✗

$= -49 \times -1 \rightarrow -49+-1 = -50$ ✗

$(x+p)(x+q) = 0$

$(x-7)(x-7) = 0$

$x-7=0$

$x=7$

d) $y^2 = -12y$

$y^2 + 12y = 0 \longrightarrow y(y+12) = 0$

y^2 $\quad$ $12y$

There is no c term, so factorise the left-hand side instead.

when you multiply two terms to get 0, one of the two terms <u>MUST</u> be 0

$y=0$ OR $y+12=0$

$y=-12$

Section B

Worked Example

Solve the quadratic equation $2y^2 - 34y - 36 = 0$ by factorisation.

Step 1: Compare the equation to the general form of a quadratic equation $ax^2 + bx + c = 0$. When $a \neq 1$, and a is a common factor of each term, factorise a out of the equation.

In this case, $a = 2$ and is a common factor of both -34 and -36:

$$2(y^2 - 17y - 18) = 0$$

Step 2: After taking out the common factor, list down the new values for a, b and c, referring to the new equation inside the bracket.

$$ax^2 + bx + c = y^2 - 17y - 18$$

Here, $a = 1,\ b = -17$ and $c = -18$.

Step 3: Write the new simplified equation in the form of $(x + p)(x + q) = 0$

In this example, our quadratic term is y^2. Hence, we write our brackets as:

$$(y + p)(y + q) = 0$$

Step 4: List out all the possible factor pairs for c.

In this example, $c = -18$. The factor pairs for -18 are:

$$\begin{array}{cc} -1 \times 18 & -2 \times 9 \\ -3 \times 6 & -6 \times 3 \\ -9 \times 2 & -18 \times 1 \end{array}$$

Step 5: Identify which factor pair adds to give b.

In this question, $b = -17$:

$$\begin{array}{cc} -1 + 18 \neq -17 & -2 + 9 \neq -17 \\ -3 + 6 \neq -17 & -6 + 3 \neq -17 \\ -9 + 2 \neq -17 & \mathbf{-18 + 1 = -17} \end{array}$$

So, the correct factor pair is 1 and -18.

Step 6: Substitute the correct factor pair as p and q in the bracket form that was formed in **Step 3**. Don't forget to include the common factor which was initially factorised out of the equation.

$$2(y + 1)(y - 18) = 0$$

Step 7: Solve the equation by setting each bracket equal to 0.

$$\begin{array}{ccc} y + 1 = 0 & \text{or} & y - 18 = 0 \\ y = -1 & & y = 18 \end{array}$$

Hence, the possible solutions for y are $\boldsymbol{y = -1}$ and $\boldsymbol{y = 18}$.

Guided Example

Solve the quadratic equation $3x^2 + 30x + 72 = 0$ by factorisation.

Step 1: Compare the equation to the general form of a quadratic equation $ax^2 + bx + c = 0$. When $a \neq 1$, and a is a common factor of each term, factorise a out of the equation.

$$3\left[\frac{3x^2}{3} + \frac{30x}{3} + \frac{72}{3}\right] = 0 \longrightarrow 3\left[x^2 + 10x + 24\right] = 0$$

Step 2: After taking out the common factor, list down the new values for a, b and c, referring to the new equation inside the bracket.

$$x^2 + 10x + 24 = 0$$

$$a = 1,\quad b = 10,\quad c = 24$$

Step 3: Write the new simplified equation in the form of $(x + p)(x + q) = 0$

$$x^2 + 10x + 24 = 0 \longrightarrow (x + p)(x + q) = 0$$

Step 4: List out all the possible factor pairs for c.

$$\begin{aligned} 24 &= 4 \times 6 \\ &= 2 \times 12 \\ &= 1 \times 24 \end{aligned}$$

Step 5: Identify which factor pair adds to give b.

$$\begin{aligned} 24 &= 4 \times 6 \longrightarrow \boxed{4 + 6} = 10 \checkmark \\ &= 2 \times 12 \longrightarrow 2 + 12 = 14 \times \\ &= 1 \times 24 \longrightarrow 1 + 24 = 25 \times \end{aligned}$$

Step 6: Substitute the correct factor pair as p and q in the bracket form that was formed in **Step 3**. Don't forget to include the common factor which was initially factorised out of the equation.

$$3(x + 4)(x + 6) = 0 \longrightarrow (3x + 12)(x + 6) = 0$$

$3x$

12

Step 7: Solve the equation by setting each bracket equal to 0.

$$\begin{aligned} 3x + 12 &= 0 \quad (-12) \\ 3x &= -12 \quad (\div 3) \\ x &= -4 \end{aligned} \qquad\qquad \begin{aligned} x + 6 &= 0 \quad (-6) \\ x &= -6 \end{aligned}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Solve the following quadratic equations by factorisation:

a) $2y^2 - 12y = 32$

$2y^2 - 12y - 32 = 0$

$2[y^2 - 6y - 16] = 0$

$-16 = -2 \times 8 \rightarrow -2 + 8 = 6$ ✗
$= -8 \times 2 \rightarrow \boxed{-8+2} = -6$ ✓
$= -16 \times 1 \rightarrow -16 + 1 = -15$
$= -1 \times 16 \rightarrow -1 + 16 = 15$

$2[(y+p)(y+q)] = 0$
$2[(y-8)(y+2)] = 0$
$(2y-16)(y+2) = 0$

$2y - 16 = 0$
$2y = 16$
$y = 8$

$y + 2 = 0$
$y = -2$

b) $\overset{3x^2}{x}(3x + 18) = 120$ $18x$

$3x^2 + 18x = 120$ -120
$3x^2 + 18x - 120 = 0$ -120

$3[x^2 + 6x - 40] = 0$

$-40 = -4 \times 10 \rightarrow \boxed{-4+10} = 6$ ✓
$= -10 \times 4 \rightarrow -10 + 4 = -6$ ✗
$= -5 \times 8 \rightarrow -5 + 8 = 3$ ✗
$= 5 \times -8 \rightarrow 5 + -8 = -3$ ✗
$=$ etc...

$3[(y+q)(y+p)] = 0$
$3[(y+10)(y-4)] = 0$
$(3y+30)(y-4) = 0$

$3y + 30 = 0$
$3y = -30$
$y = -10$

$y - 4 = 0$
$y = 4$

c) $4p^2 - 16p = \overset{8p}{2}(4p - 10)$ -20

used m and n because p is already in the question!

$4p^2 - 16p = 8p - 20$
$4p^2 - 24p + 20 = 0$
$4[p^2 - 6p + 5] = 0$

$5 = 5 \times 1 \rightarrow 5 + 1 = 6$ ✗
$= -5 \times -1 \rightarrow \boxed{-5+-1} = -6$ ✓

$4[(p+m)(p+n)] = 0$
$4[(p-5)(p-1)] = 0$
$(4p-20)(p-1) = 0$

$4p - 20 = 0$
$4p = 20$
$p = 5$

$p - 1 = 0$
$p = 1$

Section C

Worked Example

Solve the following quadratic equation by factorisation:

$$\boldsymbol{x\,(3x-17)=28}$$

Step 1: Expand and rearrange the equation in the form of ax^2+bx+c.

$$3x^2-17x-28=0$$

Step 2: When $a\neq 1$ and there is **no common factor** that can be factored out of the equation, first we need to ensure the equation is in the form of $ax^2+bx+c=0$. Then, we list down the values of a, b and c.

$$ax^2+bx+c=0$$
$$3x^2-17x-28=0$$

$$a=3,\ b=-17, \textit{ and } c=\ -28$$

Step 3: Multiply the value of a and c.

$$3\ \times -28=-84$$

Step 4: List down all the possible factor pairs of the multiplied value of a and c found in the previous step.

The factor pairs for -84 *are:*

$$\begin{array}{ccc} -1\times 84 & \textit{or} & 1\times -84 \\ -2\times 42 & \textit{or} & 2\times -42 \\ -3\times 28 & \textit{or} & 3\times -28 \\ -4\times 21 & \textit{or} & 4\times -21 \\ -6\times 14 & \textit{or} & 6\times -14 \\ -7\times 12 & \textit{or} & 7\times -12 \end{array}$$

Step 5: Identify which factor pair, when added together, gives the value of b.

In this example, $b=-17$.

The sum for the factor pairs of -84*:*

$$\begin{array}{ccc} -1+84\neq -17 & \textit{or} & 1+(-84)\neq -17 \\ -2+42\neq -17 & \textit{or} & 2+(-42)\neq -17 \\ -3+28\neq -17 & \textit{or} & 3+(-28)\neq -17 \\ -4+21\neq -17 & \textit{or} & 4+(-21)=-17 \\ -6+14\neq -17 & \textit{or} & 6+(-14)\neq -17 \\ -7+12\neq -17 & \textit{or} & 7+(-12)\neq -17 \end{array}$$

In this case, the correct factor pair would be 4 *and* -21 *since their product gives the same value as* b.

Step 6: Write the correct factor pair (found in **Step 5)** as coefficients of x, replacing bx in the original equation.

$$\mathit{Original\ Equation} \qquad \Rightarrow \quad 3x^2 - 17x - 28 = 0$$

$$\mathit{Substitute} - 17x\ \mathit{with\ the\ correct\ factor\ pair} \quad \Rightarrow \quad 3x^2 - 21x + 4x - 28 = 0$$

Step 7: For the new equation, find a common factor for the first two terms and a common factor for the last two terms. Factorise the pairs of terms separately. Ensure that the equations in the brackets are similar to one another.

$$3x^2 - 21x + 4x - 28 = 0$$

In this case, the common factor for the first two terms is $3x$ *and the common factor for the last two terms is* 4.

$$3x\,(x-7) + 4(x-7) = 0$$

Note that the brackets produced here are the same. This is how you know you've correctly performed the factorisation.

Step 8: Present the equation as the product of two brackets. The first bracket will be equal to the brackets we have formed in the previous step. The second bracket will be made up from the terms which are coefficients of the brackets in the previous step.

$$3x\,(x-7) + 4(x-7) = 0$$

$$(x-7)(3x+4) = 0$$

You can check that you have formed the brackets correctly by expanding out the brackets and checking you produce the original equation:

$$(x-7)(3x+4) = 0$$
$$3x^2 + 4x - 21x - 28 = 0$$
$$3x^2 - 17x - 28 = 0$$

Step 9: Solve the equation by comparing both factors to 0.

$$3x + 4 = 0 \qquad or \qquad x - 7 = 0$$
$$x = \frac{-4}{3} \qquad or \qquad x = 7$$

Hence, the possible solutions for x are $\boldsymbol{x = -\frac{4}{3}}$ and $\boldsymbol{x = 7}$.

Guided Example

Solve the following quadratic equation by factorisation:

$$5x^2 = 2\,(-9x + 4)$$

Step 1: Expand and rearrange the equation in the form of $ax^2 + bx + c$.

$$5x^2 = -18x + 8 \xrightarrow[+8]{-18x} 5x^2 + 18x - 8 = 0$$

Step 2: When $a \neq 1$ and there is **no common factor** that can be factored out of the equation, first we need to ensure the equation is in the form of $ax^2 + bx + c = 0$. Then, we list down the values ofa, b and c.

$$ax^2 + bx + c = 0$$
$$5x^2 + 18x - 8 = 0$$

$a = 5$
$b = 18$
$c = -8$

Step 3: Multiply the value of a and c.

$$a \times c = 5 \times -8 = -40$$

Step 4: List down all the possible factor pairs of the multiplied value of a and c found in the previous step.

$$\begin{aligned} -40 &= -5 \times 8 \text{ or } 8 \times -5 \\ &= -10 \times 4 \text{ or } 10 \times -4 \\ &= -1 \times 40 \text{ or } 1 \times -40 \\ &= -2 \times 20 \text{ or } 2 \times -20 \end{aligned}$$

Step 5: Identify which factor pair, when added together, gives the value of b.

$$\begin{aligned} -40 &= -5 \times 8 \text{ or } 8 \times -5 \\ &= -10 \times 4 \text{ or } 10 \times -4 \\ &= -1 \times 40 \text{ or } 1 \times -40 \\ &= -2 \times 20 \text{ or } 2 \times -20 \end{aligned}$$

$b = -2 + 20$

$b = 18$ ✓

Step 6: Write the correct factor pair (found in **Step 5)** as coefficients of x, replacing bx in the original equation.

$$18x = -2x + 20x$$

$$5x^2 \underline{- 2x + 20x} - 8 = 0$$

Step 7: For the new equation, find a common factor for the first two terms and a common factor for the last two terms. Factorise the pairs of terms separately. Ensure that the equations in the brackets are similar to one another.

$$5x^2 + 20x - 2x - 8 = 0$$

common factor of both pairs is $(x+4)$

$$5x^2 + 20x \rightarrow 5x(x+4)$$

$$-2x - 8 \rightarrow -2(x+4)$$

$$5x(x+4) - 2(x+4) = 0$$

Step 8: Present the equation as the product of two brackets. The first bracket will be equal to the brackets we have formed in the previous step. The second bracket will be made up from the terms which are coefficients of the brackets in the previous step.

$$5x(x+4) - 2(x+4) = 0$$

$$(5x-2)(x+4) = 0$$

Step 9: Solve the equation by comparing both factors to 0.

$$(5x-2)(x+4) = 0$$

$$5x - 2 = 0$$

$$5x = 2$$

$$\boxed{x = \frac{2}{5} = 0.4}$$

$$x + 4 = 0$$

$$\boxed{x = -4}$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

3. Solve the following quadratic equation by factorisation:

a) $6x^2 = x + 1$

$6x^2 - x - 1 = 0$

$a = 6 \quad b = -1 \quad c = -1$

$a \times c = 6 \times -1 = -6$

$-6 = -1 \times 6 \rightarrow -1 + 6 = 5$ ✗
$= -6 \times 1 \rightarrow -6 + 1 = -5$ ✗
$= -3 \times 2 \rightarrow \boxed{-3 + 2} = -1$ ✓
$= -2 \times 3 \rightarrow -2 + 3 = 1$ ✗

$-x = 2x - 3x$

$6x^2 + 2x - 3x - 1 = 0$

$2x(3x+1) - 1(3x+1) = 0$

$(2x-1)(3x+1) = 0$

$2x - 1 = 0$
$2x = 1$
$x = \frac{1}{2}$

$3x + 1 = 0$
$3x = -1$
$x = -\frac{1}{3}$

b) $4x(x - 5) = -25$

$4x^2 - 20x = -25$

$4x^2 - 20x + 25 = 0$

$a = 4 \quad b = -20 \quad c = 25$

$a \times c = 4 \times 25 = 100$

$100 = 10 \times 10 \rightarrow 10 + 10 = 20$ ✗
$= -10 \times -10 \rightarrow \boxed{-10 + -10} = -20$ ✓
$= 100 \times 1 \rightarrow 100 + 1 = 101$ ✗
$= -1 \times -100 \rightarrow -1 + -100 = -101$ ✗
$= 5 \times 20 \rightarrow 5 + 20 = 25$ ✗
$= -5 \times -20 \rightarrow -5 + -20 = -25$ ✗
= etc...

$-20x = -10x - 10x$

$4x^2 - 10x - 10x + 25 = 0$

$2x(2x-5) - 5(2x-5) = 0$

$(2x-5)(2x-5) = 0$

$2x - 5 = 0$
$2x = 5$
$x = \frac{5}{2}$

c) $26x + 20 = 6x^2$

$0 = 6x^2 - 26x - 20$

$a = 6 \quad b = -26 \quad c = -20$

$a \times c = 6 \times -20 = -120$

$-120 = 2 \times -60 \rightarrow 2 + -60 = -58$ ✗
$= 60 \times -2 \rightarrow 60 + -2 = 58$ ✗
$= 4 \times -30 \rightarrow 4 + -30 = -26$ ✓
$= 30 \times -4 \rightarrow 30 + -4 = 26$ ✗
= etc...

$-26x = -30x + 4x$

$6x^2 - 30x + 4x - 20 = 0$

$6x(x-5) + 4(x-5) = 0$

$(6x+4)(x-5) = 0$

$6x + 4 = 0$
$6x = -4$
$x = -\frac{4}{6}$
$x = -\frac{2}{3}$

$x - 5 = 0$
$x = 5$

Section D – Higher Only

Worked Example

By completing the square, express the following quadratic equation in the form of $(\boldsymbol{x}+\boldsymbol{p})^2+\boldsymbol{q}$. Then, find the exact values for x:

$$\boldsymbol{x^2-4x=\ 30}$$

Step 1: Rearrange the quadratic equation in the form of $ax^2\ +bx+c=0$. Determine the values of a, b and c.

$$x^2-4x=\ 30$$

Subtract 30 *from both sides:*

$$x^2-4x-30=\ 0$$

$$a=1,\qquad b=\ -4,\qquad c=\ -30$$

Step 2: Complete the square. Since we have $a=1$ we don't need to factorise out the leading coefficient. So completing the square can be done by substituting the b and c values into the formula $(x+\frac{b}{2})^2-\left(\frac{b}{2}\right)^2+c=0$.

$$x^2-4x-30=\ 0$$

$$(x+\frac{-4}{2})^2-\left(\frac{-4}{2}\right)^2-30=0.$$

$$(x+(-2))^2-(-2)^2-30=0.$$

$$(x-2)^2-4-30=0$$

$$(x-2)^2-34=0$$

Step 3: Solve the equation by rearranging the expression to solve for x.

$$(x-2)^2-34=0$$

$$(x-2)^2=34$$

$$x-2=\sqrt{34}$$

$$x-2=\ \pm\sqrt{34}$$

$$x=2\ \pm\sqrt{34}$$

Hence, the possible solutions for x are $\boldsymbol{x=2\ +\sqrt{34}}$ and $\boldsymbol{x=2\ -\sqrt{34}}$.

Guided Example

By completing the square, express the following quadratic equation in the form of $(x+p)^2 + q$. Then, find the exact values for x:

$$x^2 + 2 = 8x$$

Step 1: Rearrange the quadratic equation in the form of $ax^2 + bx + c = 0$. Determine the values of a, b and c.

$$x^2 + 2 = 8x$$

$-8x$ $\quad$ $-8x$

$$\underset{a}{x^2} \underset{b}{- 8x} \underset{c}{+ 2} = 0$$

$$a = 1 \quad b = -8 \quad c = 2$$

Step 2: Complete the square. Since we have $a = 1$ we don't need to factorise out the leading coefficient. So, completing the square can be done by substituting the b and c values into the formula $(x + \frac{b}{2})^2 - \left(\frac{b}{2}\right)^2 + c = 0$.

$$x^2 - 8x + 2 = 0$$

$$-\frac{8}{2} = -4$$

$$(x-4)^2 - (-4)^2 + 2 = 0$$

$$(x-4)^2 - 16 + 2 = 0$$

$$(x-4)^2 - 14 = 0$$

Step 3: Solve the equation by rearranging the expression to solve for x.

$$(x-4)^2 - 14 = 0$$

$+14$ $\quad$ $+14$

$$(x-4)^2 = 14$$

$\sqrt{\ }$ $\quad$ $\sqrt{\ }$

$$x - 4 = \pm\sqrt{14}$$

$+4$ $\quad$ $+4$

$$x = 4 \pm \sqrt{14}$$

remember to include positive AND negative solutions.

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

4. By completing the square, express the following quadratic equation in the form of $(x+p)^2+q$. Then, find the exact values for x:

a) $x^2 = -5(x+1)$

$x^2 = -5x - 5$

$\underset{a}{x^2} + \underset{b}{5x} + \underset{c}{5} = 0$

$\frac{b}{2} = \frac{5}{2}x$

$(x+\frac{5}{2})^2 - (\frac{5}{2})^2 + 5 = 0$

$(x+\frac{5}{2})^2 - \frac{25}{4} + 5 = 0$

$(x+\frac{5}{2})^2 - \frac{5}{4} = 0$

$(x+\frac{5}{2})^2 = \frac{5}{4}$

$x + \frac{5}{2} = \pm\sqrt{\frac{5}{4}}$

$x = -\frac{5}{2} \pm \sqrt{\frac{5}{4}}$

b) $x(x-6) = -4$

x^2

$-6x$

$x^2 - 6x = -4$

$x^2 - 6x + 4 = 0$

$a=1 \quad b=-6 \quad c=4$

$\frac{b}{2} = \frac{-6}{2} = -3$

$(x-3)^2 - (-3)^2 + 4 = 0$

$(x-3)^2 - 9 + 4 = 0$

$(x-3)^2 - 5 = 0$

$(x-3)^2 = 5$

$x - 3 = \pm\sqrt{5}$

$x = 3 \pm \sqrt{5}$

Section E – Higher Only

Worked Example

By completing the square, express the following quadratic equation in the form of $\boldsymbol{a(x+p)^2+q}$. Then, find the exact values for $\boldsymbol{x}$:

$$\boldsymbol{2x^2 = 1 - 4x}$$

Step 1: Rearrange the quadratic equation in the form of $ax^2 + bx + c = 0$.

$$2x^2 = 1 - 4x$$
$$2x^2 + 4x - 1 = 0$$

Step 2: Factorise the coefficient of x^2 out of the equation. Find the values a, b and c for the equation in the brackets.

$$2\left(x^2 + 2x - \frac{1}{2}\right) = 0$$
$$a = 1, \quad b = 2, \quad c = -\frac{1}{2}$$

Step 3: Complete the square for the equation in the brackets. Completing the square can be done by substituting the b and c values into the formula $(x + \frac{b}{2})^2 - \left(\frac{b}{2}\right)^2 + c = 0$.

$$2\left(x^2 + 2x - \frac{1}{2}\right) = 0$$
$$2\left[\left(x + \tfrac{2}{2}\right)^2 - \left(\tfrac{2}{2}\right)^2 - \tfrac{1}{2}\right] = 0$$
$$2\left[(x+1)^2 - 1^2 - \tfrac{1}{2}\right] = 0$$
$$2\left[(x+1)^2 - \tfrac{3}{2}\right] = 0$$
$$2\,(x+1)^2 - 3 = 0$$

Now the expression is in the form of $a(x+p)^2 + q$ where $a = 2, p = 1$ and $q = -3$.

Step 4: Solve the equation by rearranging and solving for x.

$$2\,(x+1)^2 - 3 = 0$$
$$2\,(x+1)^2 = 3$$
$$(x+1)^2 = \frac{3}{2}$$
$$x + 1 = \pm\sqrt{\frac{3}{2}}$$
$$x = \pm\sqrt{\frac{3}{2}} - 1$$

Hence, the possible solutions for x are $\boldsymbol{x = \sqrt{\frac{3}{2}} - 1}$ and $\boldsymbol{x = -\sqrt{\frac{3}{2}} - 1}$.

Guided Example

By completing the square, express the following quadratic equation in the form of $a(x+p)^2+q$. Then, find the exact values for x:

$$15x$$

$$5x\,(x+3)+9=0$$

$$5x^2$$

Step 1: Rearrange the quadratic equation in the form of $ax^2 + bx + c = 0$.

$$5x^2 + 15x + 9 = 0$$

Step 2: Factorise the coefficient of x^2 out of the equation. Find the values a, b and c for the equation in the brackets.

$$\frac{15x}{5} = 3x$$

$$5\left[x^2 + 3x + \frac{9}{5}\right] = 0$$

$$\frac{5x^2}{5} = x^2 \qquad \frac{9}{5} = \frac{9}{5}$$

$a = 1$
$b = 3$
$c = \frac{9}{5}$

Step 3: Complete the square for the equation in the brackets. Completing the square can be done by substituting the b and c values into the formula $(x+\frac{b}{2})^2 - \left(\frac{b}{2}\right)^2 + c = 0$.

$$5\left[\left(x+\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{9}{5}\right] = 0$$

$$5\left[\left(x+\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{9}{5}\right] = 0$$

$$5\left[\left(x+\frac{3}{2}\right)^2 - \frac{9}{20}\right] = 0$$

$$5\left(x+\frac{3}{2}\right)^2 - \frac{9}{4} = 0$$

Step 4: Solve the equation by rearranging and solving for x.

$$5\left(x+\frac{3}{2}\right)^2 - \frac{9}{4} = 0$$

$$5\left(x+\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\left(x+\frac{3}{2}\right)^2 = \frac{9}{20}$$

$$x+\frac{3}{2} = \pm\sqrt{\frac{9}{20}}$$

$$x = -\frac{3}{2} \pm \sqrt{\frac{9}{20}}$$

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

5. By completing the square, express the following quadratic equation in the form of $a(x+p)^2+q$. Then, find the exact value for x:

a) $4x^2 = 8x + 7$

$$4x^2 - 8x - 7 = 0$$

$$4\left[x^2 - 2x - \frac{7}{4}\right] = 0$$

$a = 1 \quad b = -2 \quad c = -\frac{7}{4}$

$\frac{b}{2} = \frac{-2}{2} = -1$

$$4\left[(x-1)^2 - (-1)^2 - \frac{7}{4}\right] = 0$$

$$4\left[(x-1)^2 - 1 - \frac{7}{4}\right] = 0$$

$$4\left[(x-1)^2 - \frac{11}{4}\right] = 0$$

$$4(x-1)^2 - 11 = 0$$

$$4(x-1)^2 = 11$$

$$(x-1)^2 = \frac{11}{4}$$

$$x - 1 = \pm\sqrt{\frac{11}{4}}$$

$$x = 1 \pm \sqrt{\frac{11}{4}}$$

b) $-3x\,(x+1) + 5 = 0$

$-3x^2$

$-3x$

$$-3x^2 - 3x + 5 = 0$$

$$3x^2 + 3x - 5 = 0$$

x-1 to make x^2 positive

$$3\left[x^2 + x - \frac{5}{3}\right] = 0$$

$a = 1 \quad b = 1 \quad c = -\frac{5}{3}$

$\frac{b}{2} = \frac{1}{2}$

$$3\left[\left(x+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - \frac{5}{3}\right] = 0$$

$$3\left[\left(x+\frac{1}{2}\right)^2 - \frac{1}{4} - \frac{5}{3}\right] = 0$$

$$3\left[\left(x+\frac{1}{2}\right)^2 - \frac{23}{12}\right] = 0$$

$$3\left(x+\frac{1}{2}\right)^2 - \frac{23}{4} = 0$$

$$3\left(x+\frac{1}{2}\right)^2 = \frac{23}{4}$$

$$\left(x+\frac{1}{2}\right)^2 = \frac{23}{12}$$

$$x + \frac{1}{2} = \pm\sqrt{\frac{23}{12}}$$

$$x = -\frac{1}{2} \pm \sqrt{\frac{23}{12}}$$

Section F – Higher Only

Worked Example

Solve the quadratic equation $x^2 = 5x + 8$ by using the quadratic formula. Give your answer for x to two decimal places.

Step 1: Ensure the given equation is in the form of $ax^2 + bx + c = 0$. Determine the values of a, b and c.

$$\boldsymbol{x^2 = 5x + 8}$$
$$x^2 - 5x = 8$$
$$x^2 - 5x - 8 = 0 \quad \Longrightarrow \quad a = 1, b = -5, c = -8$$

Step 2: Substitute the value of a, b and c into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-8)}}{2(1)} = \frac{5 \pm \sqrt{25 + 32}}{2} = \frac{5 \pm \sqrt{57}}{2}$$

$x = \frac{5+\sqrt{57}}{2}$ *or* $x = \frac{5-\sqrt{57}}{2}$

$x = 6.27$ $\quad$ $x = -1.27$

Hence, the possible solutions for x are $\boldsymbol{x = 6.27}$ and $\boldsymbol{x = -1.27}$.

Guided Example

Solve the quadratic equation $9x + x^2 = 11$ by using the quadratic formula. Give your answer for x to two decimal places.

Step 1: Ensure the given equation is in the form of $ax^2 + bx + c = 0$. Determine the values of a, b and c.

$$x^2 + 9x - 11 = 0$$

$a = 1 \quad b = 9 \quad c = -11$

Step 2: Substitute the value of a, b and c into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-9 \pm \sqrt{9^2 - 4 \times 1 \times -11}}{2 \times 1} = \frac{-9 \pm \sqrt{81 - (-44)}}{2} = \frac{-9 \pm \sqrt{125}}{2}$$

$$\frac{-9 + \sqrt{125}}{2} = 1.090 \qquad \frac{-9 - \sqrt{125}}{2} = -10.090$$

$\therefore \quad x = 1.09$

$x = -10.09$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

6. Solve the following quadratic equation by using the quadratic formula. Give your answer for x to two decimal places:

a) $(x + 2)^2 = x + 16$

$x^2 + 4x + 4 = x + 16$

$x^2 + 3x - 12 = 0$

$a = 1 \quad b = 3 \quad c = -12$

$x = \dfrac{-3 \pm \sqrt{3^2 - 4 \times 1 \times -12}}{2 \times 1} = \dfrac{-3 \pm \sqrt{9 - (-48)}}{2} = \dfrac{-3 \pm \sqrt{57}}{2}$

$\dfrac{-3 + \sqrt{57}}{2} = 2.275 \qquad \dfrac{-3 - \sqrt{57}}{2} = -5.275$

$\therefore x = 2.27$
$x = -5.27$

b) $10x - 9 = 3 - 2x^2 - 7x$

$2x^2 + 10x - 9 = 3 - 7x$

$2x^2 + 17x - 12 = 0$

$a = 2 \quad b = 17 \quad c = -12$

$x = \dfrac{-17 \pm \sqrt{17^2 - 4 \times 2 \times -12}}{2 \times 2} = \dfrac{-17 \pm \sqrt{289 - (-96)}}{4} = \dfrac{-17 \pm \sqrt{385}}{4}$

$\dfrac{-17 + \sqrt{385}}{4} = 0.655 \qquad \dfrac{-17 - \sqrt{385}}{4} = -9.155$

$\therefore x = 0.66$
$x = -9.16$

c) $(x + 3)(x - 1) = 1 + 4x(x - 2)$

$x^2 + 2x - 3 = 1 + 4x^2 - 8x$

$2x - 3 = 1 + 3x^2 - 8x$

$0 = 3x^2 - 10x + 4$

$a = 3 \quad b = -10 \quad c = 4$

$x = \dfrac{-(-10) \pm \sqrt{10^2 - 4 \times 3 \times 4}}{2 \times 3} = \dfrac{10 \pm \sqrt{100 - 48}}{6} = \dfrac{10 \pm \sqrt{52}}{6}$

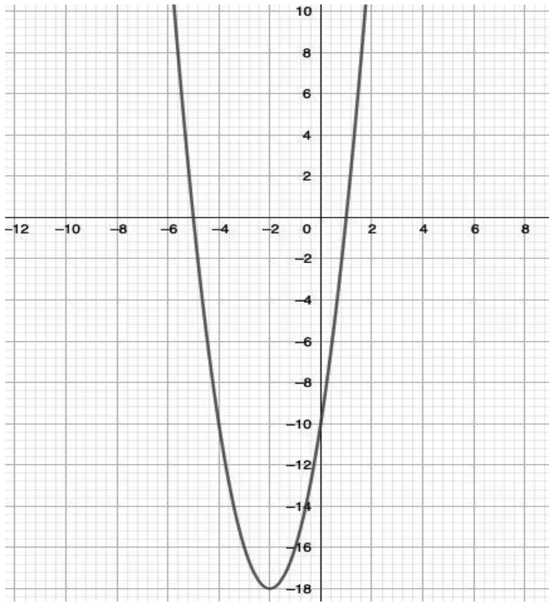
$\dfrac{10 + \sqrt{52}}{6} = 2.869 \qquad \dfrac{10 - \sqrt{52}}{6} = 0.465$

$\therefore x = 2.87$
$x = 0.46$

Section G

Worked Example

Given the graph $y = 2x^2 + 8x - 10$, find both coordinates of the x –intercepts.



Step 1: Ensure the quadratic equation is in the form of $ax^2 + bx + c$. For x –intercepts, $y = 0$. Hence, we can substitute the value of y with 0 into the given equation.

$$2x^2 + 8x - 10 = 0$$

Step 2: Solve the equation to find possible values of x.

We solve the quadratic by factorisation:

$$2\,(x^2 + 4x - 5) = 0$$

$$2\,(x - 1)(x + 5) = 0$$

Equate the factors to 0 to solve for x:

$$2\,(x - 1)(x + 5) = 0$$

$x - 1 = 0$ *or* $x + 5 = 0$
$x = 1$ *or* $x = -5$

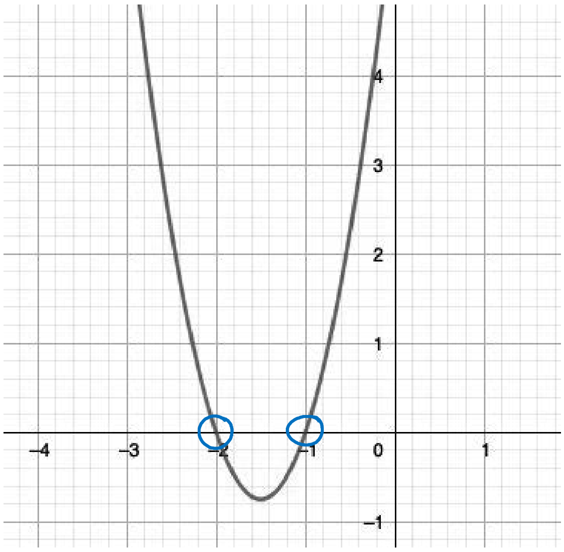
Hence, the coordinates of the x –intercept are $(1,0)$ and $(-5,0)$.

Step 3: We can check if these answers are correct from the given graph.

With reference to the graph above, we can observe that the line intersects the x –axis at point $(1,0)$ and $(-5,0)$. Hence, the answers are correct.

Guided Example

Given the graph $y - 6 = 3x^2 + 9x$, find both coordinates of the x –intercepts.



Step 1: Ensure the quadratic equation is in the form of $ax^2 + bx + c$. For x –intercepts, $y = 0$. Hence, we can substitute the value of y with 0 into the given equation.

$$y = 3x^2 + 9x + 6 \longrightarrow 0 = 3x^2 + 9x + 6$$

Step 2: Solve the equation to find possible values of x.

$$\div 3 \quad 3x^2 + 9x + 6 = 0 \quad \div 3$$
$$x^2 + 3x + 2 = 0$$
$$2 = 2 \times 1 \rightarrow \boxed{2+1} = 3 ✓$$
$$= -2 \times -1 \rightarrow -2-1 = -3 \times$$
$$(x+2)(x+1) = 0$$
$$x+2=0 \qquad x+1=0$$
$$x=-2 \qquad x=-1$$

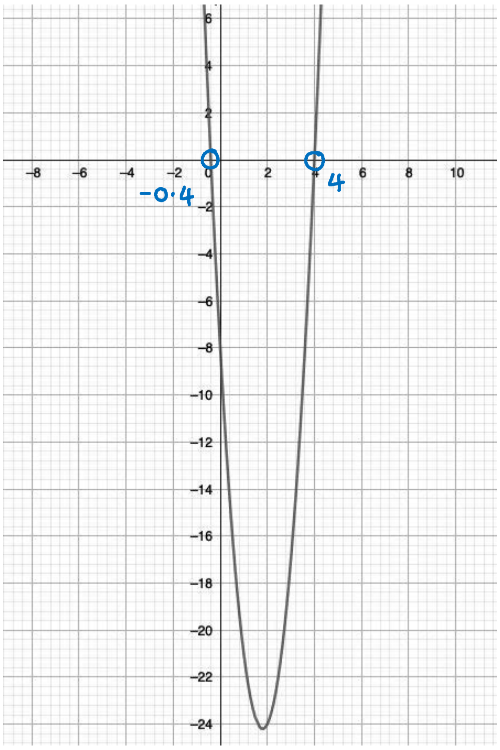
Step 3: We can check if these answers are correct from the given graph.

Graph intersects x-axis (when y is 0) at $x=-2$ and $x=-1$, therefore these answers are correct.

Now it's your turn!

If you get stuck, look back at the worked and guided examples.

7. Given the graph below, find both coordinates of the x –intercepts. The equation of the graph is $y + 18x + 8 = 5x^2$.



$y + 18x + 8 = 5x^2$

$y = 5x^2 - 18x - 8$ ← from inspection we can see that this won't factorise easily, so complete the square or use quadratic formula.

$a=5 \quad b=-18 \quad c=-8$

$$x = \frac{-(-18) \pm \sqrt{(-18)^2 - 4\times5\times-8}}{2\times5}$$

$$= \frac{18 \pm \sqrt{324 - (-160)}}{10}$$

$$= \frac{18 \pm \sqrt{484}}{10}$$

$$= \frac{18 \pm 22}{10}$$

$\frac{18+22}{10} = 4$ and $\frac{18-22}{10} = -0.4$

These correspond to intercepts on graph therefore:

$x = -0.4 \quad x = 4$